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Heavy Quark Cascade Mechanism of Tetramuon Production by Neutrinos in a Scale-Breaking Model

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A few neutrino-induced tetramuon events have recently been reported by two experimental groups with a production rate of $\sim 10^{-6}$ for $\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+)/\sigma(\nu \to \mu^-)$. However, the rate for such events is not yet determined theoretically. In the present paper, we report on a detailed calculation of the rate in the framework of a heavy quark cascade mechanism, using the QCD-improved parton distribution functions parametrized by Buras and Gaemers, which have successfully fitted various experimental data. Our calculation for $\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+)/\sigma(\nu \to \mu)$ accords well with the CDHS and FHPRW rate. The computed rate is also in accord with that obtained via the radiative charm model calculation, whereby the normal charm dimuon production is accompanied by a radiative $\mu^+\mu^-$ pair.

1. INTRODUCTION

A number of tetralepton events have been reported: two $\nu N \to \mu^-\mu^-\mu^+\mu^+ X$ from the CDHS (Holder et al., 1978) and FHPRW (Cline and Ling, 1978) groups and one $\bar{\nu} N \to \mu^+ e^+ e^-$ from the BHFSW group (Lubatti, 1978). The CDHS rate $\sigma(\nu \to 4\mu)/\sigma(\nu \to \mu)$ is $\sim 10^{-6}$, within a factor of 10 anyway. A new tetramuon event has also been reported by the FHOPRW Collaboration (Gilchreise, 1978; Peyaird, 1978; Loveless, 1978). The rate for such events is not determined.

Restricting ourselves to the consideration of tetramuon events only, we report in this paper a detailed calculation of the production rates

$$\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+ X)/\sigma(\nu \to \mu^- X)$$

and

$$\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+ X)/\sigma(\nu \to \mu^-\mu^+ X)$$

in the framework of a heavy quark cascade mechanism, using the QCD-improved parton density functions parametrized by Buras and Gaemers (Buras and Gaemers, 1977) which have successfully fitted various experimental data. The prediction of the model for the rate is compared with the available CDHS and FHOPRW data and with the estimates of the rates expected from other sources of neutrino-induced tetramuon events such as the following: (i) the production of charm with radiation making an additional $\mu^+\mu^-$ pair (the radiative charm model); (ii) charm with an extra hadronic muon pair or a cc^- pair; (iii) the heavy lepton cascade model (the LC model).

The paper is organized as follows: In Section 2, we present the quark cascade model of the observed neutrino-induced right-sign tetramuoun events and derive cross sections and production rates, in the scale-violating QCD model of Buras and Gaemers. In Section 3, we present numerical estimates of the production rates

$$\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+X)/\sigma(\nu \to \mu^-X)$$

and

$$\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+X)/\sigma(\nu \to \mu^-\mu^+X)$$

In Section 4, we give our summary.

2. QUARK CASCADE MECHANISM OF TETRAMUON PRODUCTION BY NEUTRINOS

According to this mechanism, the neutrino-induced production of the top quark t is followed by a decay chain

$$t \to b \to c \to s \tag{2.1}$$

giving rise to a right-sign tetramuon signal. A schematic representation of this process

$$\begin{aligned} &\nu_\mu + N \to \mu^- + t \cdots \\ &\qquad\qquad\qquad \searrow b + \mu^+ + \nu \cdots \\ &\qquad\qquad\qquad\qquad\qquad \searrow c + \mu^- + \bar{\nu} \\ &\qquad\qquad\qquad\qquad\qquad\qquad \searrow \mu^+ + s + \nu \cdots \end{aligned} \tag{2.2}$$

is shown in Figure 1.

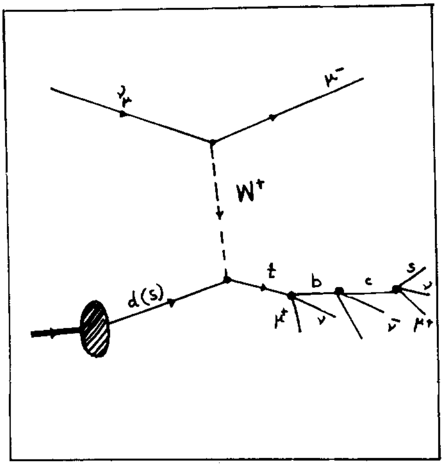

Fig. 1. t-decay chain in inclusive neutrino charged current interaction.

Production of the Top Quark. We now consider the production of the top quark by neutrinos, in the Kobayashi–Maskawa (Kobayashi and Maskawa, 1973) model, which is a simple model in which the t quark can be produced by the neutrino, via a V–A charged current coupling. The model, which is a simple extension of the Cabibbo–GIM model (Cabibbo, 1964; Glashow, Iliopoulous, and Maiani, 1970) is formed from six quarks. Apart from the neutrino masses vanishing in the model, the model supports parity violation in atomic physics, for which there is now ample evidence (Barkov and Zolotorev, 1978; Sherden, 1978), thereby removing another motivation for the introduction of right-handed couplings. The model also incorporates CP violation in a way that is consistent with low-energy phenomenology (Pakvassa and Sagawara, 1976; Maiani, 1976, Ellis, Gaillard, and Nanapoulos, 1976).

The V–A charged-current coupling is given by

$$L_{cc} = g_\mu^{W^+} q_L \gamma_\mu \tfrac{1}{2} \tau^- O q_L + \text{H.c.} \tag{2.3}$$

where O is a 3×3 generalized Cabibbo matrix. The quark vectors in (2.3) are given by

$$q_L = \begin{bmatrix} d \\ s \\ b \end{bmatrix}_L \tag{2.4}$$

and

$$\bar{q}_L = (\bar{u}, \bar{c}, \bar{t})_L \tag{2.5}$$

The O matrix is defined by

$$O = \begin{bmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 + s_2s_3e^{i\delta} & c_1c_2s_3 - s_3c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3 - c_2s_3e^{i\delta} & c_1s_2s_3 + c_2c_3e^{i\delta} \end{bmatrix} \tag{2.6}$$

where $s_i = \sin\theta_i$, $c_i = \cos\theta_i$, $i = 1, 2, 3$, and θ_1, θ_2, θ_3 and δ are mixing angles; $\delta \neq 0$ leads to CP violation. The mixing angles are small. If $s_i \ll 1$, for all the mixing angles, then the matrix O simplifies to

$$O \simeq \begin{bmatrix} 1 & s_1 & s_1s_3 \\ -s_1 & 1 & s_3 - s_2e^{i\delta} \\ -s_1s_2 & s_2 - s_3e^{i\delta} & 1 \end{bmatrix} \tag{2.7}$$

We can easily identify θ_1 with θ_c, the usual Cabibbo angle. The empirical limits of s_2 and s_3 are given by

$$s_3^2 < 0.06, s_2^2 \lesssim 0.1 \tag{2.8}$$

(Ellis et al. 1976, 1977). Thus, all $\sin^2\theta_i$ are less than 0.1. When we discount the possibility of a strong cancellation between s_2 and s_3 in the elements $s_3 - s_2e^{i\delta}$ and $s_2 - s_3e^{i\delta}$, in the matrix O of equation (2.7), we obtain

$$O \simeq \begin{bmatrix} 1 & \sin\theta_c & \sin\theta_c\sin\theta_3 \\ -\sin\theta_c & 1 & \sin\theta_3 \\ -\sin\theta_2\sin\theta_c & \sin\theta_2 & 1 \end{bmatrix} \tag{2.9}$$

The heavy-quark charged-current V–A couplings then give

$$\begin{aligned} J_\mu^- = {} & \bar{u}\gamma_{,\mu}(1+\gamma_5)d + \sin\theta_c\bar{u}\gamma_\mu(1+\gamma_5)s \\ & + \sin\theta_3\sin\theta_c\bar{u}\gamma_\mu(1+\gamma_5)b + \sin\theta_c\bar{c}\gamma_\mu(1+\gamma_5)d \\ & + \bar{c}\gamma_\mu(1+\gamma_5)s + \sin\theta_3\bar{c}\gamma_\mu(1+\gamma_5)b \\ & + \sin\theta_2\sin\theta_c\bar{t}\gamma_\mu(1+\gamma_5)d + \sin\theta_2\bar{t}\gamma_\mu(1+\gamma_5)s + \bar{t}\gamma_\mu(1+\gamma_5)b \end{aligned} \tag{2.10}$$

The current couplings relevant to the production of the top quark in the final state (neglecting transitions from one heavy quark to another heavy quark (Kaplan and Martin, 1976)) is given by

$$J_\mu^-\left[\nu d(s)\right]\rightarrow\mu^- t) = \sin\theta_2 \sin\theta_c \bar{t}\gamma_\mu(1+\gamma_5)d + \sin\theta_2 \bar{t}\gamma_\mu(1+\gamma_5)s \quad (2.11)$$

In units of G^2ME/π, the t-production differential cross section is given by

$$\frac{d\sigma}{dy\,dz_t}(\nu\rightarrow\mu - t) = z_t\left\{\sin^2\theta_2 \sin^2\theta_c\left[u(z_t)+d(z_t)\right] + 2s(z_t)\alpha\cdot\sin^2\theta_2\right\} Y_t^+ \quad (2.12)$$

where

$$z_t = x + \frac{m_t^2}{2MEy} \quad (2.13)$$

is the slow-rescaling variable, $u(z_t)$ the u-quark density above threshold for producing the top quark, etc.,

$$Y_t^+ = 1 - \frac{m_t^2}{2MEz_t} \quad (2.14)$$

and the parameter $\alpha(0 \leqslant \alpha < 1)$ is the percentage contribution of the sea quarks relative to the valence quarks.

In the scale-breaking model where the quark distribution functions depend on x (or z) and Q^2, the square of the transfered four-momentum, the differential cross section for the production of the top quark modifies to

$$\frac{d\sigma}{dy\,dz_t}(\nu\rightarrow\mu^- t) = \frac{G^2ME}{\pi} z_t\left\{\sin^2\theta_c \sin^2\theta_2\left[u(z_t, Q^2) + d(z_t, Q^2)\right]\right.$$
$$\left. + \alpha\cdot 2s(z_t, Q^2)\sin^2\theta_2\right\} Y_t^+ \quad (2.15)$$

The Scale-Breaking Model of Buras and Gaemers. We now introduce a model of scaling violation which uses the QCD-improved quark distribution functions parametrized by Buras and Gaemers. The approximations for the sea and valence quark densities are respectively given by

$$x\xi(x,Q^2)=\xi_1(Q^2)\left[\frac{\xi_1(Q^2)}{\xi_2(Q^2)}-1\right](1-x)^{\left[\frac{\xi_1(Q^2)}{\xi_2(Q^2)}-2\right]} \tag{2.16}$$

$$x\xi^c(x,Q^2)=\xi_1^c(Q^2)\left[\frac{\xi_1^c(Q^2)}{\xi_2^c(Q^2)}-1\right](1-x)^{\left[\frac{\xi_1^c(Q^2)}{\xi_2^c(Q^2)}-2\right]} \tag{2.17}$$

and

$$xv(x,Q^2)=\frac{1.5x^{0.7-0.176\ln L}(1-x)^{2.6+0.8\ln L}}{B(0.7-0.176\ln L,3.6+0.8\ln L)} \tag{2.18}$$

where the beta function in the denominator of equation (2.18) is to ensure the zeroth moment sum rule, and $\xi_1(Q^2),\xi_2(Q^2)$ are the Q^2-dependent first and second moments of the sea quark densities, while $\xi_1^c(Q^2)$ and $\xi_2^c(Q^2)$ are the Q^2-dependent first and second moments of the charmed quark distribution functions. The Q^2-dependent first and second moments of the valence quark distribution functions are represented by $v_1(Q^2)$ and $v_2(Q^2)$, respectively. These moments are given as follows.

(a) First Moments:

$$v_1(Q^2)=v_1L^{-32/75} \tag{2.19}$$

$$\xi_1(Q^2)=\tfrac{1}{4}\left[\tfrac{3}{14}+\left(3\xi_1+v_1+\xi_1^c-\tfrac{3}{14}\right)L^{-56/75}\right.$$
$$\left.+(\xi_1-\xi_1^c-v_1)L^{-32/75}\right] \tag{2.20}$$

$$\xi_1^c(Q^2)=\tfrac{1}{4}\left[\tfrac{3}{14}+\left(3\xi_1+v_1+\xi_1^c-\tfrac{3}{14}\right)L^{-56/75}\right.$$
$$\left.-(3\xi_1-3\xi_1^c+v_1)L^{-32/75}\right] \tag{2.21}$$

where the moments on the right-hand side are evaluated at a moderately low $Q^2(Q^2\approx Q_0^2)$ and

$$L=\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \tag{2.22}$$

Λ being the scale-breaking parameter and Q_0 a reference four-momentum at whose value the scaling quark distribution functions are extracted by fitting low-energy neutrino data. We take $Q_0^2{\sim}1.8$ GeV.

(b) Second Moments. These are given by

$$v_2(Q^2) = v_2 L^{-2/3} \tag{2.23}$$

$$\xi_2(Q^2) = \tfrac{1}{4}\left[O_1^2(Q^2) + (\xi_2 - \xi_2^c - v_2)L^{-2/3}\right] \tag{2.24}$$

$$\xi_2^c(Q^2) = \tfrac{1}{4}\left[O_1^2(Q^2) - (3\xi_2 - 3\xi_2^c - v_2)L^{-2/3}\right] \tag{2.25}$$

where the singlet operator O_1^2 is given by

$$\begin{aligned} O_1^2(Q^2) = &\left[0.925(3\xi_2 + \xi_2^c + v_2)\right.\\ &\left.+0.144 G_2\right]L^{-0.609}\\ &+\left[0.075(3\xi_2 + \xi_2^c - v_2) - 0.144 G_2\right]L^{-1.386} \end{aligned} \tag{2.26}$$

G_2 being the second gluon moment. We define the valence quark v, by

$$2v = v_u + v_d$$

The $SU(3)$ symmetric sea is represented by ξ, while the charm component on the sea is denoted by ξ^c. In our calculations we shall use the maximal value of the scale-breaking parameter allowed by the electron and muon data; i.e., $\Lambda = 0.5$. We shall also use the value of 0.057 for G_2.

Input Moments. We calculate the input moments ξ_1, v_1, ξ_2, v_2, of the theory from the Barger–Phillips parametrization of quark density functions (Barger and Phillips, 1974) obtained from fits to SLAC data (Bodek et al., 1973) on

$$\nu W_2^{ep} \equiv F_2^{ep}, \qquad \nu W_2^{en} \equiv F_2^{en}$$

The parametrization is given by

$$v_u(x) = 0.594x^{-1/2}(1-x^2)^3 + 0.461x^{-1/2}(1-x^2)^5 + 0.612x^{-1/2}(1-x^2)^7 \tag{2.27}$$

$$v_d(x) = 0.072x^{-1/2}(1-x^2)^3 + 0.206x^{-1/2}(1-x^2)^5 + 0.621x^{-1/2}(1-x^2)^7 \tag{2.28}$$

$$\xi(x) = 0.145x^{-1}(1-x)^9 \tag{2.29}$$

$$\xi^c(x) = 0.014\xi(x) \tag{2.30}$$

We can also denote the up-and-down valence quark densities by the following:

$$u(x) = v_u(x) + \xi(x) \tag{2.31}$$

$$d(x) = v_d(x) + \xi(x) \tag{2.32}$$

For the sea quarks, we have

$$\bar{u}(x) = \bar{d}(x) = s(x) = \bar{s}(x) = \xi(x) \tag{2.33}$$

The computed input moments then give

$$\begin{aligned} &\xi_1 \simeq 0.015 \\ &\xi_2 \simeq 0.0015 \\ &v_1 \simeq 0.213 \\ &v_2 \simeq 0.098 \end{aligned} \tag{2.34}$$

The Q^2-dependent moments are then calculated to give

$$\begin{aligned} \xi_1(Q^2) \simeq 0.0535 + 0.018\left(\ln\frac{Q^2}{0.25}\right)^{-0.7467} \\ -0.066\left(\ln\frac{Q^2}{0.25}\right)^{-0.4267} \end{aligned} \tag{2.35}$$

$$v_1(Q^2) \simeq 0.248\left(\ln\frac{Q^2}{0.25}\right)^{-0.4267} \tag{2.36}$$

$$\begin{aligned} \xi_1^c(Q^2) \simeq 0.0535 + 0.018\left(\ln\frac{Q^2}{0.25}\right)^{-0.7467} \\ -0.08\left(\ln\frac{Q^2}{0.25}\right)^{-0.467} \end{aligned} \tag{2.37}$$

The Q^2-dependent quark distribution function becomes

$$x\xi(x,Q^2) \simeq \left[0.0535 + 0.018\left(\ln\frac{Q^2}{0.25}\right)^{-0.7467} - 0.066\left(\ln\frac{Q^2}{0.25}\right)^{-0.426}\right]$$

$$\times\left\{\frac{\begin{array}{c}0.0535+0.018[\ln(Q^2/0.25)]^{-0.7467}\\ -0.066[\ln(Q^2/0.25)]^{-0.4267}\end{array}}{\begin{array}{c}0.039[\ln(Q^2/0.25)]^{-0.609}+0.055[\ln(Q^2/0.25)]^{-1.386}\\ -0.038[\ln(Q^2/0.25)]^{-0.67}\end{array}}-1\right\}$$

$$\times(1-x)^{P_1}$$

where

$$P_1=\left\{\frac{\begin{array}{c}0.0535+0.018[\ln(Q^2/0.25)]^{-0.7467}\\ -0.066[\ln(Q^2/0.25)]^{-0.4267}\end{array}}{\begin{array}{c}0.039[\ln(Q^2/0.25)]^{-0.609}+0.055[\ln(Q^2/0.25)]^{-1.356}\\ -0.038[\ln(Q^2/0.25)]^{-0.67}\end{array}}-2\right\}$$

(2.38)

$$xv(x,Q^2) \simeq \frac{1.5x^{0.7-0.09\ln(Q^2/0.25)}(1-x)^{2.6+0.41\ln(Q^2/0.25)}}{B(0.7-0.9\ln(Q^2/0.25), 3.6+0.4\ln(Q^2/0.25))} \quad (2.39)$$

3. NUMERICAL ESTIMATES AND TETRAMUON PRODUCTION RATES

In order to obtain numerical estimates for cross sections and tetramuon production rates relative to the single-muon and dimuon rates, one needs, in addition to the parametrization of the quark density functions, estimates of the masses of the heavy quark (charm, top, and bottom), the mixing angles $\theta_c, \theta_2, \theta_1$, the parameter α, and the bounds on the variables x, y, z_t, z_b, z_c, etc. In this connection, we shall adopt the following estimates for the quantities:

$$\begin{aligned}
&m_c \simeq 0.15\ \text{GeV}, \qquad m_t \simeq 8\,\text{GeV}, \qquad m_b \simeq 5\,\text{GeV}\\
&\sin^2\theta_3 \lesssim 0.06\\
&\sin^2\theta_2 \simeq 0.1, \qquad \cos^2\theta_2 \simeq 0.94, \qquad \cos^2\theta_c \simeq 0.95\\
&\sin^2\theta_c \simeq 0.05
\end{aligned} \tag{3.1}$$

Bounds on the Variables. In the scale-violating quark model, the effect of Q^2 should be taken into account in determining the integration limits of the variables of interest. For a heavy quark t, in the final state, for example, one obtains the following bounds:

$$\frac{Q_0^2}{2ME} \leqslant x \leqslant 1 - \frac{m_t^2}{2ME} \tag{3.2}$$

$$\frac{Q_0^2 + m_t^2}{2ME} \leqslant y_t \leqslant 1 \tag{3.3}$$

$$\frac{Q_0^2 + m_t^2}{2ME} \leqslant z_t \leqslant 1 \tag{3.4}$$

Finally, we provide estimates of the branching ratios of heavy quarks. The branching ratio of $t \to b$ is expected to be very sensitive to the mass of the top quark. The following estimates of the branching ratios of semi-leptonic decays of heavy quarks (c, t, b) are available in the literature (Ellis et al., 1977; Ali and Pietarinen, 1979; Holder et al., 1977):

$$\text{Br}(b \to \bar{c} + \bar{\mu} + \bar{\nu}_\mu + \text{hadrons}) \simeq 0.3 \tag{3.5}$$

$$\text{Br}(t \to b + \mu^+ + \nu) \simeq 0.2 \tag{3.6}$$

$$\text{Br}(c \to \mu^+ + s + \nu) \simeq 0.15 \tag{3.7}$$

We shall limit our calculations to neutrino energies in the range $\sim 100 \sim 150$ GeV, and to high Q^2 values ($Q^2 > 4$ GeV).

We now integrate equation (2.12) for $E \sim 100\text{–}150$ GeV and $Q^2 > 4$ GeV, to obtain the total cross section for the production of the top quark t, by the neutrino. The values of the cross section for values of E and Q^2 within the stated range and for $\alpha = 0.1$ are provided in Table I.

TABLE I. t-Production Cross Section

E_ν(GeV)	Q^2(GeV)	$\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)$
100	15	$\sim\frac{G^2ME}{\pi}(0.000104)$
150	15	$\sim\frac{GME}{\pi}(0.000235)$
100	20	$\sim\frac{G^2ME}{\pi}(0.0000959)$
150	20	$\sim\frac{G^2ME}{\pi}(0.0002)$

Neutrino-Induced Charged Current Single-Muon and Dimuon Cross Sections above Charm Threshold. We require neutrino-induced single-muon cross section above charm threshold as well as the dimuon cross section, both in the scale-violating QCD model of Buras and Gaemers, in order to estimate the right-sign tetramuon production rates relative to the single-muon and dimuon rates in inclusive neutrino scattering. The single-muon and dimuon differential cross sections in the GIM scheme are given by (Chukwumah, 1980)

$$\frac{d\sigma}{dy\,dx}(\mu^-)=\frac{G^2ME}{\pi}\left\{x(2v(x,Q^2)\cos^2\theta_c+\alpha\cdot\left[1+(1-y)^2\right]2\xi(x,Q^2)\right.$$
$$\left.+z_c\left[2v(z_c,Q^2)\sin^2\theta_c+\alpha\cdot 2\xi(z_c,Q^2)\right](1-B_\nu^c)\right\} \tag{3.8}$$

$$\frac{d\sigma}{dz_c\,dy}(\mu^-\mu^+)=\frac{G^2ME}{\pi}\cdot z_c\left[2v(z_c,Q^2)\sin^2\theta_c+\alpha\cdot 2\xi(z_c,Q^2)\right]B_\nu^c \tag{3.9}$$

where

$$z_c=x+\frac{m_c^2}{2MEy} \tag{3.10}$$

B_ν^c is the branching ratio of the charmed quark to the muon.

We can now proceed to estimate right-sign tetramuon production rates. The right-sign tetramuon total cross section is estimated as

$$\begin{aligned}\sigma(\mu^-\mu^-\mu^+\mu^+)\sim\sigma(\nu\to\mu^-t)&\mathrm{Br}(t\to b+\mu^++\nu)\\ &\times\mathrm{Br}(b\to c+\mu^-+\bar{\nu})\\ &\times\mathrm{Br}(c\to\mu^++s+\nu)\end{aligned} \tag{3.11}$$

TABLE II. Right-Sign Tetramuon Production Rates: $\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-)$ and $\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-\mu^+)$

E_ν(GeV)	Q^2(GeV)	$\frac{\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)}{\sigma(\nu\to\mu^-)}$	$\frac{\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)}{\sigma(\nu\to\mu^-\mu^+)}$
100	15	$\sim 5.2\times 10^{-6}$	$\sim 5.3\times 10^{-4}$
	20	$\sim 5.19\times 10^{-6}$	$\sim 5.28\times 10^{-4}$
150	15	$\sim 1.16\times 10^{-5}$	$\sim 1.16\times 10^{-3}$
	20	$\sim 1.07\times 10^{-5}$	$\sim 1.07\times 10^{-3}$

From equations (3.11), (3.5), (3.6), and (3.7), we obtain

$$\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+X)\sim 0.009\sigma(\nu\to\mu^-t) \tag{3.12}$$

Integrating equations (3.8) and (3.9) for $E\sim 100$–150 GeV, $\alpha=0.1, Q^2>4$ GeV, to obtain the total cross sections $\sigma(\nu\to\mu^-)$ and $\sigma(\nu\to\mu^-\mu^+)$, and using equations (3.12) and Table I, we obtain the right-sign tetramuon rates

$$\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-)$$

and

$$\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-\mu^+)$$

These rates are summarized in Table II.

4. SUMMARY

We have carried out the quark cascade calculations for the ratios

$$\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-)$$

and

$$\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-\mu^+)$$

using the QCD-improved quark densities parametrized by Buras and Gaemers. We have worked in the neutrino energy range ~ 100–150 GeV and for $Q^2>4$ GeV, while the sea quark contribution relative to the valence contribution is taken as little as 10%. Our model predicts that the two rates $\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-)$ and $\sigma(\nu\to\mu^-\mu^-\mu^+\mu^+)/\sigma(\nu\to\mu^-\mu^+)$ fall slightly with increasing neutrino energy and the square of the transferred four-momentum, Q^2. Also the calculated rate $\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+)/\sigma(\nu \to \mu^-)$ is in accord with the estimated rate of 10^{-7}–10^{-5}, suggested in the literature (Cline, 1978), which is itself in agreement with the CDHS and FHPRW experimental data (Phillips, 1978). The quark cascade model prediction for $\sigma(\nu \to \mu^- \mu^+ \mu^+)/\sigma(\nu \to \mu^-)$ is also in accord with the lepton cascade estimate of $\sim 10^{-5}$, for $\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+)/\sigma(\nu \to \mu^-)$, though in a fancy scheme (De Rujula et al., 1978) with an extra lepton and right-handed couplings.

The radiative charm model estimate (Barger, Gottschalk, and Phillips, 1978) gives a rate

$$\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+)/\sigma(\nu \to \mu^-) \sim 10^{-7}$$

averaged over the CDHS and FHPRW spectrum with $E > 100$ GeV. This rate is within the limits of the quark cascade calculations.

However, the quark cascade rate is much higher than the triple charm prediction for $\sigma(\nu \to \mu^-\mu^-\mu^+\mu^+)/\sigma(\nu \to \mu^-)$, which in the lowest-order QCD gives a small rate $\sim 10^{-9}$, averaging over the CDHS and FHPRW spectrum with $E \sim 100$ GeV.

We conclude that the quark cascade mechanism, wrought in the scale-violating QCD model of Buras and Gaemers, is as good a candidate as the radiative charm mechanism and the heavy lepton cascade mechanism, in explaining the origin of right-sign tetramuon events recently observed in neutrino scattering at high energies. Nevertheless, more tetramuon events are needed before their source/sources can be regarded as permanently established.

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